

COMPUTATIONAL METHODS FOR THE AERODYNAMIC DESIGN

Constantin ROTARU*, Ionică CÎRCIU**, Mircea BOȘCOIANU**

*Military Technical Academy, Bucharest, Romania

**"Henri Coandă" Air Force Academy, Brasov, Romania

Abstract: The interest for efficient methods for aerodynamic design is expressed by a better use of the panel method applications, based on the property that no volumetric grid is required. In this case, only the bounding surface should be discretized. This paper deals with the Maple soft applications for numerical panel method for incompressible flows. Based on this powerful mathematical computation tools, several numerical solutions were obtained for different body shapes, in a simple but intuitive manner.

Keywords: the panel method, boundary elements, Maple soft applications, flow simulation.

1. INTRODUCTION

Despite advances in the development of sophisticated grid generation techniques and compressible flow modeling methods, analyses based on incompressible and inviscid potential flow assumptions continue to remain a very good technique for aerodynamic computation.

Compared to the generation of a good quality volumetric grid, construction of a surface mesh for a complex geometry is a simpler problem especially since the required geometric information is often already available in the form of CAD files and efficient grid generation techniques for curved surfaces have been developed.

The incompressible potential flow model provides reliable flowfield predictions over a wide range of conditions. For the potential flow assumption to be valid for aerodynamics calculations, the primary requirement is that viscous effects are small in the flowfield, and that the flowfield must be subsonic everywhere. If the local flow is at such a low speed everywhere that it can be assumed incompressible, Laplace's Equation is an exact representation of the inviscid flow. For higher subsonic mach numbers with small disturbances to the freestream flow, the Prandtl-Glauert Equation can be used and converted to Laplace's Equation by a simple

transformation. This provides the basis for estimating the initial effects of compressibility on the flowfield, namely linearized subsonic flow. In both cases, the flowfield can be found by the solution of a single linear partial differential equation. One of the key features of Laplace's Equation is the property that allows the equation governing the flowfield to be converted from 3D problem throughout the field a 2D problem for finding the potential on the surface.

The solution is then found using this property by distributing singularities of unknown strength over discretized portions of the surface: panels. Hence the flowfield solution is found by representing the surface by a number of panels and solving a linear set of algebraic equations to determine the unknown strengths of the singularities.

2. PROBLEM FORMULATION

The equation for the potential is the following

$$\varphi = V_{\infty}(x \cos \alpha + y \sin \alpha) + \sum_{j=1}^N \int_{\text{panel } j} \left[\frac{q_j(s)}{2\pi} - \frac{\gamma_j \theta}{2\pi} \right] \quad (1)$$

q_j is the 2D source strength vortex singularity of strength $\gamma_j(s)$

where V_∞ is the free stream velocity, α the angle of attack and $\theta = \tan^{-1}(y/x)$.

The numbering system starts at the lower surface trailing edge and proceeds forward, around the leading edge and aft to upper surface trailing edge, so $N + 1$ points define N panels. The approach is to break up the surface into straight line segments, assume the source strength $q(s)$ is constant over each line segment (panel), but with different value for each panel, $q(s) = q_j, j = 1, 2, \dots, N$, and the vortex strength γ is constant and equal over each panel.

Most airfoils are not described by equations but are defined by a table of coordinates. Frequently, these coordinates are the results of a computational aerodynamic design program, and simple algebraic formulas can not be used to define the shape. The representation of a smooth surface by a series of line segments is presented in Fig. 1.

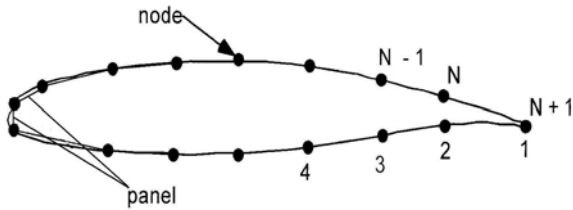


Fig. 1 Representation of nodes

If the i^{th} panel is between the i^{th} and $i+1^{\text{th}}$ nodes and the i^{th} panel's inclination to the x axis is θ_i (Fig. 2), then the sin and cos of θ are given by

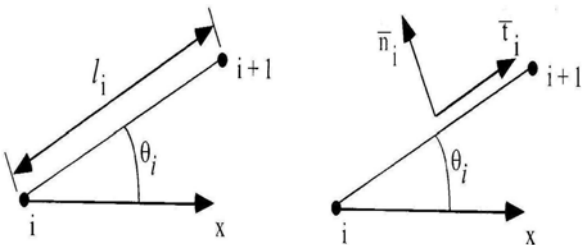


Fig. 2 Nomenclature for coordinate system

$$\begin{cases} \sin \theta_i = \frac{y_{i+1} - y_i}{l_i} \\ \cos \theta_i = \frac{x_{i+1} - x_i}{l_i} \end{cases} \quad (2)$$

and the normal and tangential unit vectors are

$$\begin{cases} \vec{n}_i = -\sin \theta_i \vec{i} + \cos \theta_i \vec{j} \\ \vec{t}_i = \cos \theta_i \vec{i} + \sin \theta_i \vec{j} \end{cases} \quad (3)$$

The unknowns have to satisfy the flow tangency condition on each panel at one specific control point and the solution has to satisfy the Kutta condition. The control point is the mid-point of each panel as shown in Fig. 3.

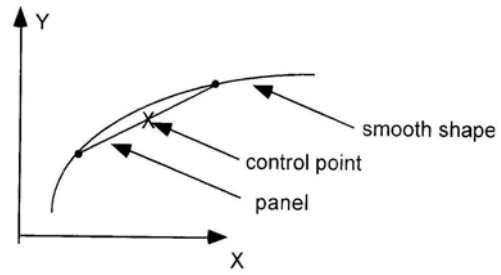


Fig. 3 Control point representation

The coordinates of the control point are

$$\begin{cases} \bar{x}_i = \frac{x_i + x_{i+1}}{2} \\ \bar{y}_i = \frac{y_i + y_{i+1}}{2} \end{cases} \quad (4)$$

and the velocity components at the control point (\bar{x}_i, \bar{y}_i) are $u_i = u(\bar{x}_i, \bar{y}_i)$ and $v_i = v(\bar{x}_i, \bar{y}_i)$.

The flow tangency boundary condition is given by $\vec{V} \cdot \vec{n} = 0$,

$$(u_i \vec{i} + v_i \vec{j}) \cdot (-\sin \theta_i \vec{i} + \cos \theta_i \vec{j}) = 0 \quad (5)$$

or

$$-u_i \sin \theta_i + v_i \cos \theta_i = 0$$

for each $i, i = 1, 2, \dots, N$.

The remaining relation is found from the Kutta condition. This condition states that the flow must leave the trailing edge smoothly. Because of the importance of the Kutta condition in determining the flow, the solution is extremely sensitive to the flow details at the trailing edge, and for this reason the last panels on the top and bottom are small and of equal length.

The trailing edge panel nomenclature is presented in Fig. 4.

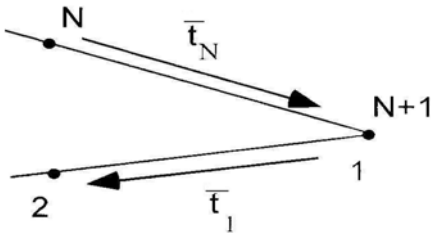


Fig. 4 Trailing edge

Also, the magnitude of the tangential velocities on the upper and lower surface is equal

$$u_{t1} = -u_{tN}$$

or

$$\vec{V} \cdot \vec{t}_1 = -\vec{V} \cdot \vec{t}_N \quad (6)$$

Carrying out the operation one gets the relation

$$\begin{aligned} (u_1 \vec{i} + v_1 \vec{j}) \cdot (\cos \theta_1 \vec{i} + \sin \theta_1 \vec{j}) = \\ - (u_N \vec{i} + v_N \vec{j}) \cdot (\cos \theta_N \vec{i} + \sin \theta_N \vec{j}) \end{aligned} \quad (7)$$

which is expanded to obtain the final relation:

$$\begin{aligned} u_1 \cos \theta_1 + v_1 \sin \theta_1 = \\ - u_N \cos \theta_N + v_N \sin \theta_N \end{aligned} \quad (8)$$

The expression for the potential in terms of the singularities on each panel and the boundary conditions derived above for the flow tangency and Kutta condition are used to construct a system of linear algebraic equations for the strengths of the sources and the vortex. The velocity components at any point i are given by contributions from the velocities induced by the source and vortex distributions over each panel,

$$\begin{cases} u_i = V_\infty \cos \alpha + \sum_{j=1}^N q_j u_{sij} + \gamma \sum_{j=1}^N u_{vij} \\ v_i = V_\infty \sin \alpha + \sum_{j=1}^N q_j v_{sij} + \gamma \sum_{j=1}^N v_{vij} \end{cases} \quad (9)$$

where q_j and γ are the singularity strengths and the u_{sij} , v_{sij} , u_{vij} and v_{vij} are the influence coefficients (for instance, the influence coefficient u_{sij} is the x-component of velocity at x_i due to a unit source distribution over the j^{th} panel).

In order to find u_{sij} , v_{sij} , u_{vij} and v_{vij} it is necessary to work in a local panel coordinate system (x^p, y^p) which leads to a straightforward means of integrating source and vortex distributions along a straight line segment. The system is locally aligned with each panel j , and is connected to the global coordinate system, like in Fig. 5.

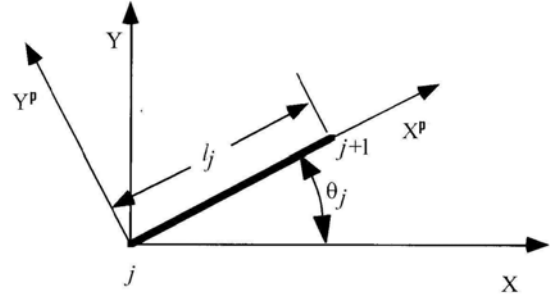


Fig. 5 Local panel nomenclature

The influence coefficients determined in the local coordinate system aligned with a particular panel are $u^{(p)}$ and $v^{(p)}$ and are transformed back to the global coordinate system by

$$\begin{cases} u = u^{(p)} \cos \theta_j - v^{(p)} \sin \theta_j \\ v = u^{(p)} \sin \theta_j + v^{(p)} \cos \theta_j \end{cases} \quad (10)$$

The velocity field induced by a source in its Cartesian coordinates (taking into account that the source is located at the origin, $r = 0$) is

$$\begin{cases} u(x, y) = \frac{Q}{2\pi} \frac{x}{x^2 + y^2} \\ v(x, y) = \frac{Q}{2\pi} \frac{y}{x^2 + y^2} \end{cases} \quad (11)$$

If the source is located along the x-axis at a point $x = t$, the velocities induced by the source distributions are

$$\begin{aligned} u_s &= \frac{1}{2\pi} \int_{t=0}^{t=1} \frac{q(t)}{2\pi} \frac{x-t}{(x-t)^2 + y^2} \\ v_s &= \frac{1}{2\pi} \int_{t=0}^{t=1} \frac{q(t)}{2\pi} \frac{y}{(x-t)^2 + y^2} \end{aligned} \quad (12)$$

To obtain the influence coefficients, one can write u_s and v_s in the local panel

coordinate system with $q(t)=1$ (unit source strength):

$$u^{(p)}_{sij} = -\frac{1}{2\pi} \ln\left(\frac{r_{i,j+1}}{r_{ij}}\right) \quad (13)$$

$$v^{(p)}_{sij} = \frac{v_1 - v_0}{2\pi} = \frac{\beta_{ij}}{2\pi}$$

where r_{ij} is the distance from the j^{th} node to the point i , which is taken to be the control point location of the i^{th} panel. The angle β_{ij} is the angle subtended at the middle of the i^{th} panel by the j^{th} panel (Fig. 6).

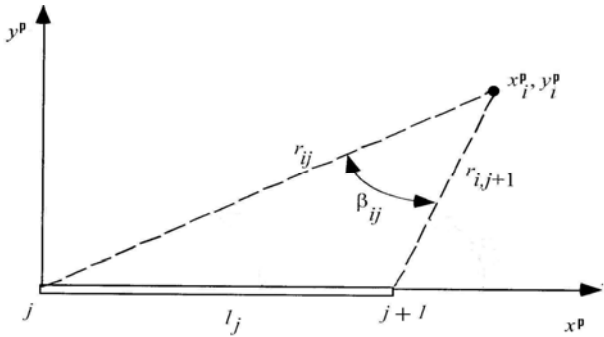


Fig. 6 Angles and radius

The source induces normal velocities and no tangential velocities. Thus, $u^{(p)}_{sii} = 0$ and $v^{(p)}_{sii}$ depends on the side from which the panel control point is approached. Approaching the panel control point from the outside leads to $\beta_{ii} = \pi$, while approaching from the inside leads to $\beta_{ii} = -\pi$.

Based on the same analysis used for the source singularities one can get the formulas for the influence coefficients due to the vortex distribution

$$u^{(p)}_{vij} = +\frac{1}{2\pi} \int_{t=0}^{t=l_j} \frac{y_i^{(p)}}{(x_i^{(p)} - t)^2 + y_i^{(p)2}} dt = \frac{\beta_{ij}}{2\pi}$$

$$v^{(p)}_{vij} = \frac{1}{2\pi} \int_{t=0}^{t=l_j} \frac{x_i^{(p)} - t}{(x_i^{(p)} - t)^2 + y_i^{(p)2}} dt = \frac{1}{2\pi} \ln\left(\frac{r_{i,j+1}}{r_{ij}}\right) \quad (14)$$

where the definitions and special circumstances described for the source singularities are the same in the current case of distributed vortices. In this case the vortex

distribution induces an axial velocity on itself at the sheet and no normal velocity.

In order to get a system of equations of the form

$$\sum_{j=1}^N A_{ij} q_j + A_{i,N+1} \gamma = b_i \quad i = 1, 2, \dots, N \quad (15)$$

which are solved for unknown source and vortex strengths, it is necessary to write the flow tangency conditions

$$-u_i \sin \theta_i + v_i \cos \theta_i = 0 \quad (16)$$

for each i , $i = 1, 2, \dots, N$, where the velocities are given by:

$$u_i = V_\infty \cos \alpha + \sum_{j=1}^N q_j u_{sij} + \gamma \sum_{j=1}^N u_{vij} \quad (17)$$

$$v_i = V_\infty \sin \alpha + \sum_{j=1}^N q_j v_{sij} + \gamma \sum_{j=1}^N v_{vij}$$

Taking into account formulas for u_{sij} , v_{sij} , u_{vij} , v_{vij} , one can get the system

$$\begin{cases} A_{ij} = \frac{1}{2\pi} \ln\left(\frac{r_{i,j+1}}{r_{i,j}}\right) \sin(\theta_i - \theta_j) + \frac{\beta_{ij}}{2\pi} \cos(\theta_i - \theta_j) \\ A_{i,N+1} = \frac{1}{2\pi} \sum_{j=1}^N \left[\ln\left(\frac{r_{i,j+1}}{r_{i,j}}\right) \cos(\theta_i - \theta_j) - \beta_{ij} \sin(\theta_i - \theta_j) \right] \\ b_i = V_\infty \sin(\theta_i - \alpha) \end{cases}$$

The final equations associated with the Kutta condition are

$$\begin{cases} A_{N+1,j} = \frac{1}{2\pi} [\sin(\theta_1 - \theta_j) \beta_{1,j} + \sin(\theta_N - \theta_j) \beta_{N,j}] - \\ - \frac{1}{2\pi} \left[\cos(\theta_1 - \theta_j) \ln\left(\frac{r_{1,j+1}}{r_{1,j}}\right) + \cos(\theta_N - \theta_j) \ln\left(\frac{r_{N,j+1}}{r_{N,j}}\right) \right] \\ A_{N+1,N+1} = \sum_{j=1}^N \left[\sin(\theta_1 - \theta_j) \ln\left(\frac{r_{1,j+1}}{r_{1,j}}\right) + \sin(\theta_N - \theta_j) \right. \\ \cdot \left. \ln\left(\frac{r_{N,j+1}}{r_{N,j}}\right) + \sum_{j=1}^N [\cos(\theta_1 - \theta_j) \beta_{1,j} + \cos(\theta_N - \theta_j) \beta_{N,j}] \right] \\ b_{N+1} = -V_\infty \cos(\theta_1 - \alpha) - V_\infty \cos(\theta_N - \alpha) \end{cases}$$

The coefficients derived above provide the required coefficients to solve a system of

linear algebraic equations for $N + 1$ unknowns q_i , $i = 1, 2, \dots, N$ and γ ,

$$\begin{cases} \sum_{j=1}^N A_{ij} q_j + A_{i, N+1} \gamma = b_i & i = 1, 2, \dots, N \\ \sum_{j=1}^N A_{N+1, j} q_j + A_{N+1, N+1} \gamma = b_{N+1} \end{cases} \quad (18)$$

At each control point, $v_n = 0$ and the tangential velocity is

$$\begin{aligned} u_{t_i} = & V_\infty \cos(\theta_i - \alpha) + \\ & + \frac{1}{2\pi} \sum_{j=1}^N \left[\beta_{ij} \sin(\theta_i - \theta_j) - \ln\left(\frac{r_{i, j+1}}{r_{i, j}}\right) \cos(\theta_i - \theta_j) \right] q_j + \\ & + \frac{\gamma}{2\pi} \sum_{j=1}^N \left[\ln\left(\frac{r_{i, j+1}}{r_{i, j}}\right) \sin(\theta_i - \theta_j) + \beta_{ij} \cos(\theta_i - \theta_j) \right] \end{aligned}$$

and the pressure coefficient,

$$C_{p_i} = 1 - \left(\frac{u_{t_i}}{V_\infty} \right)^2 \quad (19)$$

3. MAPLE SOLUTION

Maple provides an interactive environment for solving symbolic, numeric and graphical computations. A simple way to solve the system of equations for q_i , $i = 1, 2, \dots, N$ and γ , is to build a procedure **proc**(V_∞, α) which gives the results for the inputs: free stream velocity V_∞ and angle of attack α . After declaring the local and global variables one can choose the number of panels, N and write the coordinates of nodes i , where $i = 1, 2, \dots, N + 1$. The matrix M of coefficients A_{ij} and vector B of b_i , are written in the following form:

$$M := \text{Matrix}([\text{seq}(L[i], I = 1 \dots N + 1)])$$

where

$$L[i] := [\text{seq}(A[i, j], j = 1 \dots N + 1)]$$

And

$$B := \text{Vector}([\text{seq}(b[i], I = 1 \dots N + 1)])$$

With the package `LinearSolve` one can get the solution of the equation

$$A \cdot x = B \quad (20)$$

namely,

$$x = [q_1, q_2, \dots, q_N, \gamma] \quad (21)$$

and after that, the tangential velocity u_{t_i} at each control point and the pressure coefficient C_{p_i} .

In the following figures are presented the NACA 4415 airfoil (Fig. 7) and some results for C_{p_i} (Fig. 8, Mach number 0.2, Fig. 9, Mach number 0.4).

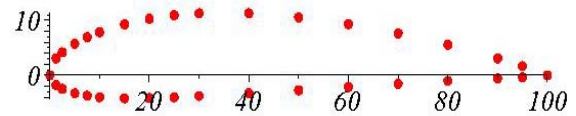


Fig. 7 Airfoil coordinates

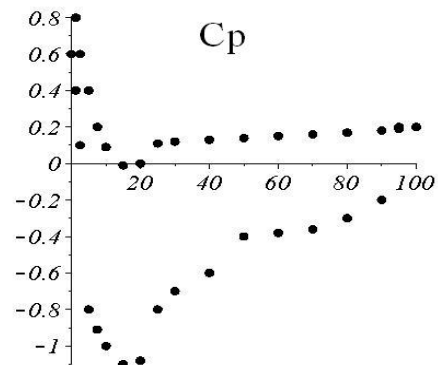


Fig. 8 Pressure coefficient

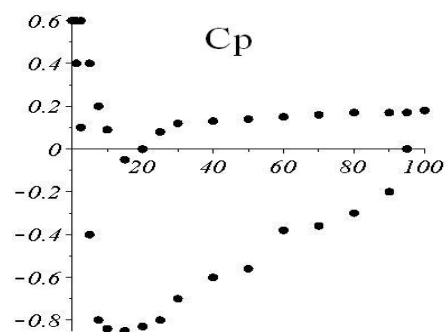


Fig. 9 Pressure coefficient

Also, a very simple way to solve the system of equations for the numerical source panel method (analyzed very well in the *Fundamentals of Aerodynamics* – fourth edition by John D. Anderson Jr.) is allowed by Maple soft.

For the flow around the circular cylinder, the equation for the source panel strengths $\lambda_1, \lambda_2, \dots, \lambda_N$ is:

$$\frac{\lambda_i}{2} + \sum_{\substack{j=1 \\ (j \neq i)}}^N \frac{\lambda_j}{2\pi} \int_{j \text{ panel}} \frac{\partial}{\partial n_i} (\ln r_{ij}) ds_j + V_\infty \cos \beta_i = 0$$

The velocity tangent to the surface at each control point can be calculated as a sum of the contribution from the freestream and from the source panels,

$$V_{ti} = V_\infty \sin \beta_i + \sum_{j=1}^N \frac{\lambda_j}{2\pi} \int_{j \text{ panel}} \frac{\partial}{\partial s} (\ln r_{ij}) ds_j$$

where β_i is the angle between V_∞ and normal vector to the panel i .

In Fig. 10 are presented the control points (1, 2, ..., 12) for source panel distribution around a circular cylinder.

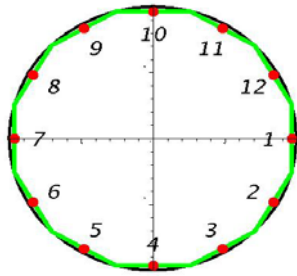


Fig. 10 Control points representation

Results for the pressure coefficients obtained from Maple calculation (red points in Fig. 11) are compared with the exact analytical result (black line). The numerical pressure coefficient result is placed on the analytical line, that is the maple calculation is right.

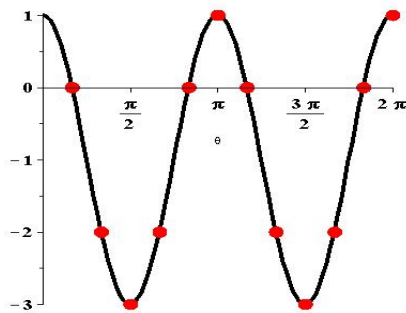


Fig. 11 Numerical and analytical results

4. CONCLUSIONS

The linear nature of the potential flow allows the principle of superposition in order to construct solutions to problems of interest by summing simpler solutions. Flows due to sources, doublets and vortices are the elementary building blocks out of which general solutions are constructed. Conventional computational fluid dynamics (CFD) methods require calculation for the entire three-dimensional field about the body, while the panel method also can calculate the entire three-dimensional field but it requires only calculation over the surface of the body.

One major computational task in a panel method is to obtain the matrices of panel influences at each others' control points. The key formulas are obtained by integrating over a panel the point source and doublet formulas weighted by the proper polynomial variations. These integrations, which express the panel influences at a general field point in space, may be performed analytically over a plane panel to obtain exact closed form expressions, which however are rather complicated. Maple software is an efficient tool capable to solve different types of problems and to explore mathematical models for computational aerodynamics.

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